resistors provide reasonably good terminations at least to 5 GHz. The performance of the chip termination further documents the case for placing any transducer of this type directly across the slot.

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Analytic Model for Varactor-Tuned Waveguide Gunn Oscillators

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Abstract—**An analytic model for electronic tuning of an X-band waveguide transferred-electron oscillator is presented. The oscillator is electronically tunable by a varactor, and mechanically tunable by movement of a short circuit. The model is used to predict oscillation frequency, maximum electronic tuning range, and electronic tuning versus varactor bias voltage. Two different methods, the "zero reactance theory" and the Slater perturbation theory, are used to calculate the electronic tuning. The results of these calculations are compared to experimental results for two different oscillator configurations.**

INTRODUCTION

The objective of this short paper is a theoretical prediction of both the mechanical and electronic tuning characteristics for a varactor-tuned oscillator using a transferred-electron device operated CW and mounted in full-height X-band waveguides. Wide electronic tuning ranges can be obtained using coaxial structures [1] or reduced-height waveguides [2]. When low FM noise is desired, higher Q structures involving waveguide cavities with their associated smaller tuning ranges are useful.

The input data for the calculations reported consist of the dimensions of the waveguide and mounting posts together with the usually specified varactor and Gunn diode parameters, including device package parameters for both. In this short paper we present an analytical model, and compare calculations based on this model using two different theories, with experimental data. Several Gunn diodes and varactors have been used in two oscillator configurations with typical results reported here.

ANALYTIC MODEL

The basis for the calculations in this paper is an extension of a previously reported model used to predict the mechanical tuning of a waveguide-mounted Gunn device, with and without a coupling iris [3]. In this equivalent circuit representation the mounting post is represented by the Marcuvitz theory for a finite diameter inductive post [4], while the capacitive gap in which the device is placed is included using the theory of Eisenhart and Khan [5].

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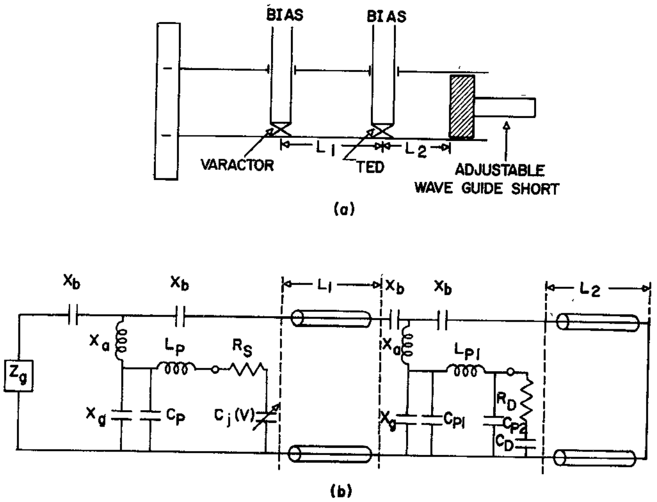

Fig. 1. (a) Oscillator configuration for case 1; case 2 is obtained by interchanging the varactor and Gunn device. The waveguide inside dimensions are 0.900 × 0.400 in; the two posts have a diameter of 0.120 in. (b) The equivalent circuit. X_a and X_b represent the post(s) as given by the Marcuvitz theory; X_g is the gap reactance from Eisenhart and Khan [5]. C_p and L_p are varactor package parameters; R_s is the varactor series resistance; and C_j is the junction capacitance. C_{p1}, C_{p2}, and L_{p1} are Gunn device package parameters; C_D and R_D are to represent tne GaAs chip. The Gunn devices for the data shown here are Microwave Associates MA49156; the varactors are MA45103. L_1 and L_2 are lengths of waveguide with characteristic impedance $Z_g(\omega)$.

In this paper we add an additional mounting post for the varactor, together with the varactor and associated package parameters, to the previously described model. The resulting equivalent circuit is shown in Fig. 1. The varactor and Gunn diode parameters are those specified by the commercial manufacturers.

Computations are performed with a digital computer using two different methods to determine the change in frequency due to a change in varactor bias.

In the first method, which we call the "zero reactance theory" [3]–[7] the center frequency of the Gunn oscillator is calculated using a search for frequencies such that the conditions corresponding to stable circuit-controlled oscillations are met, namely,

$$X_T(\omega) = X_D(\omega) + X_L(\omega) = 0 \tag{1}$$

$$\partial X_T/\partial \omega > 0 \tag{2}$$

where $X_D(\omega)$ is the reactance of the Gunn device at the oscillation frequency and $X_L(\omega)$ is the load reactance seen by the Gunn device.

Electronic tuning is calculated as a shift in oscillation frequency as the varactor dc bias is varied. This, of course, necessitates expressing the varactor junction capacitance as a function of bias voltage, such that the change in load reactance seen by the Gunn device may be calculated for a change in varactor bias voltage.

One of several differences between these calculations and those previously reported [7] is the retention of the varactor series resistance R_s. This loss element affects the range of electronic tuning, and is especially significant for the calculation of electronic tuning using our second method which is based on Slater perturbation theory [8].

SLATER PERTURBATION THEORY

The Slater perturbation theory is often applied in techniques for determining electromagnetic field configurations in resonant structures by introducing a perturbing volume and measuring the resultant shift in resonant frequency. In our second method for calculating electric tuning, the varactor may be thought of as a perturbing volume within the larger oscillator resonator. It can then be shown that (see Appendix)

$$\frac{\delta f}{f} \simeq \frac{Q_v \delta P_v + P_v \delta Q_v}{(P_0 + P_v) Q_L} \tag{3}$$

where Q_v is the varactor Q given by

$$Q_v = 1/\omega R_s C_j. \tag{5}$$

C_j is the varactor junction capacitance, and Q_L is the loaded Q of the cavity, including the energy stored in the Gunn device and the varactor. P_v is the power dissipated in the varactor, and P_0 is the power delivered to the waveguide load. (Losses in the cavity walls have been neglected.) The Slater perturbation theory is employed by evaluating the change in varactor power dissipation and varactor Q corresponding to changes in varactor bias voltage; P_v is evaluated in the context of the circuit model of Fig. 1 as the power dissipated in R_s.

The perturbation theory holds only for small changes in energy. Thus a calculation of electronic tuning based on a reasonably large change in varactor voltage is possible if the calculation proceeds by an addition of small changes [9]. A small voltage increment is applied and the resultant Δf evaluated from (4); this is repeated N times to obtain the resultant electronic tuning as

$$\Delta f = \sum_{n=1}^{n} \Delta f_n. \tag{6}$$

These calculations, involving the details of changes in varactor RF voltage and current and junction capacitance provide an additional test of the usefulness of this, and similar methods of modeling a post-mounted device.

OSCILLATOR CONFIGURATIONS

The oscillators considered here each consist of a section of waveguide in which are located two centrally positioned mounting posts. One end of the waveguide is connected to a sliding short circuit; the other is terminated in a matched load. Both mounting posts have bias insertion capability; the devices are mounted at one end of a post, against a broad wall of the waveguide. The two different oscillator configurations described here are achieved by exchanging the position of the Gunn and varactor devices between the two mounting posts. Thus, case 1 is for the Gunn device mounted on the post closest to the short circuit, while case 2 has the varactor in this position.

The Gunn diode and package parameters used are: $C_D = 0.2$ pF; $R_D = -4\ \Omega$; $L_{P1} = 6.5 \times 10^{-10}$ h; $C_{P1} = 0.1$ pF; and $C_{P2} = 0.12$ pF. The parameters associated with the varactor diode are: $L_P = 4.2 \times 10^{-10}$ h; $Q = 2.5 \times 10^3$; and

$$C_j(V) = \frac{3.179 \times 10^{-12}}{(0.6 + V_{\text{applied}})} 0.45.$$

RESULTS

In case 1 the center frequency of the oscillator is primarily determined by L_2, the spacing between the Gunn device mounting post and the short; in case 2 the center frequency is primarily determined by L_1, the spacing between the Gunn device mount and the varactor mounting post. Case 1 is mechanically tunable over a wide range, while case 2 is not.

In this paper we report a wide mechanical tuning range for case 1 (Fig. 2) without the occurrence of mode jumping, a condition in which the oscillation resonance switches from the $\lambda_g/2$ to the λ_g waveguide mode as L_2 is increased. The stability against mode jumping was such that the power varied from 6 to 0.5 mW with mechanical tuning from about 12.0 to 8.5 GHz without a mode switch occurring. Case 1 is similar to the iris-coupled oscillator previously reported [3], in that the oscillator frequency is primarily determined by the position of the sliding short circuit, while the tuning characteristic is modified by the reactance of the post-mounted varactor.

The variation in center frequency as a function of waveguide short-circuit position for case 2 is shown in Fig. 3. The center frequency is found to vary only slightly as a function of L_2 (compared to case 1) since the oscillator center frequency is primarily determined by the varactor-to-Gunn diode spacing of 1.02 in. (This spacing approximately corresponds to $\lambda_g/2$ at the center frequency.)

Electronic tuning is shown by plotting Δf versus varactor voltage. This is shown for both oscillator configurations in Fig. 4 where experimental results are compared to theoretical predictions based on both (1) and (2) and the perturbation theory (4). Good agreement

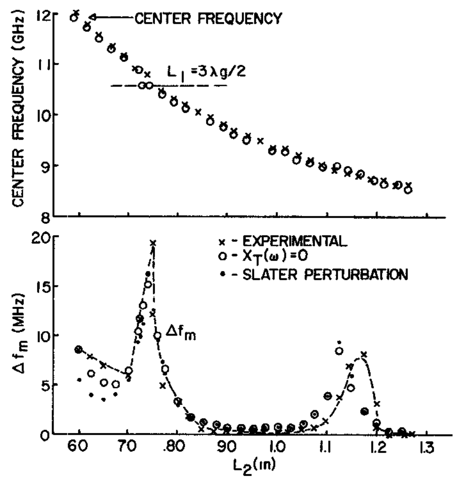

Fig. 2. Center frequency (mechanical tuning) and Δf_m (electronic tuning range corresponding to change in varactor bias from 4 to 45 V) for case 1. L_1 is 2.25 in.

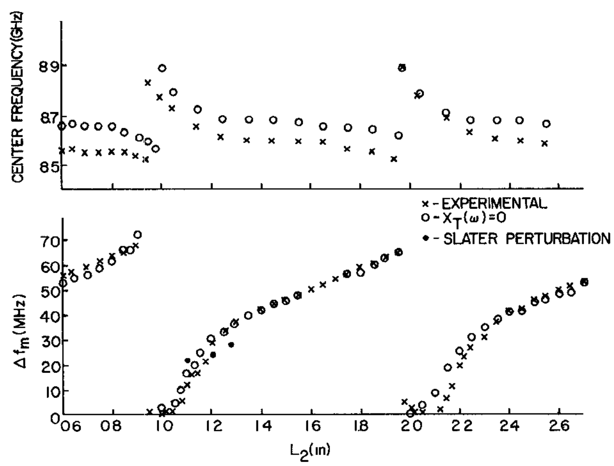

Fig. 3. Center frequency (mechanical tuning) and Δf_m (electronic tuning range corresponding to change in varactor bias from 4 to 45 V) for case 2. L_1 is 1.02 in. The perturbation theory was evaluated for only three values of L_2 as agreement with experiment was poor and the expenditure of additional computer time seemed unnecessary.

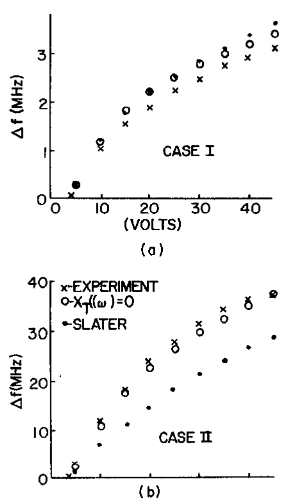

Fig. 4. (a) Electronic tuning (Δf versus varactor bias voltage for bias from 4 to 45 V) for case 1. $L_1 = 2.25$ in; $L_2 = 0.8$ in; $f = 9.62$ GHz at 4 V (negative bias). (b) Electronic tuning for case 2. $L_1 = 1.02$ in; $L_2 = 1.275$ in; $f = 8.55$ GHz at 4 V (negative bias).

was generally found for case 1; the agreement for case 2 was not as good, as seen in Fig. 4(b). In fact, for arbitrary values of L_2 (the short-circuit-to-varactor distance) case 2 is usually characterized by better agreement between experimental data and (1) and (2), than with the perturbation theory. In general, the perturbation theory predictions may be in error by as much as a factor of 2 for case 2, while calculations based on (1) and (2) are within 20 percent.

Of great interest in design of a given cavity configuration is the maximum available electronic tuning. This may be examined as the parameter Δf_m, which is defined as the change in frequency corresponding to a change in varactor bias voltage from 4 to 45 V. This quantity is plotted for case 1 in Fig. 2, and for case 2 in Fig. 3, both as a functions of L_2. Examining case 1, first we see that Δf_m varies considerably with L_2. The peaks in Δf_m occur where L_1 and L_2 are both approximate multiples of $\lambda_g/2$ at the oscillation frequency. Fig. 2 also shows several points calculated using the perturbation theory, indicating good agreement with experiment.

Fig. 3 shows Δf_m versus L_2 for case 2. One observes discontinuities in the characteristics of both center frequency and Δf_m versus L_2 these occur for L_2 values corresponding to λ_g and $\lambda_g/2$. At these values of L_2 the calculations based on (1) and (2) show two distinct resonances with relatively similar frequencies; the oscillator is always observed to "jump" to the resonance with the higher Q.

The experimental curves were all taken with L_2 increasing; some hysteresis is generally observed [3] when instead L_2 is decreasing.

CONCLUSION

It has been shown that an analytic model is available which provides good predictions of electronic tuning for wide variations in varactor voltage and oscillator dimensions. This includes Δf_m and also Δf as a function of varactor bias voltage. The theory accurately predicts certain frequency jumps which are therefore primarily due to the circuit elements as modeled.

The usual use of (1) and (2) for oscillator frequency prediction has been augmented by use of the Slater perturbation theory with good results.

APPENDIX

One may consider a cavity including regions of positive and negative conductivity representing varactor loss and negative differential mobility, respectively. Then the conventional formulation for the Slater perturbation theorem may be modified to give

$$\iint_S (H \times E_0^* + H_0^* \times E) \cdot dS$$

$$= \int_{\tau V} \sigma_v E \cdot E_0^* \, d\tau_v + \int_{\tau_s} \sigma_s E \cdot E_0^* \, d\tau_s + j \int_V (\omega - \omega_0) \cdot [\epsilon E \cdot E_0^* + \mu H \cdot H_0^*] \, dV + j \int_V \omega \delta \epsilon E \cdot E_0^* \, dV \qquad (7)$$

where S represents the surface of the resonant cavity including the output port; V is the region surrounded by S including the regions τ_v and τ_s, representing the varactor and the transferred-electron oscillator, respectively, where $\sigma_v > 0$ and $\sigma_s < 0$.

For small perturbations, one assumes

$$E \cdot E_0^* \simeq |E_0|^2$$

$$H \cdot H_0^* \simeq |H_0|^2$$

and we have the oscillator output power given by

$$P_0 = -\frac{1}{2} \iint_S (H_0 \times E_0^* + H_0^* \times E_0) \cdot dS$$

while the power dissipated in the varactor is

$$P_V = \frac{1}{2} \int \sigma_V |E_0|^2 d\tau_V.$$

The loaded Q is defined such that

$$Q_L = \omega \left(\frac{1}{2} \int_V [\epsilon |E_0|^2 + \mu |H_0|^2] \, dV \right) \Big/ (P_0 + P_V). \qquad (8)$$

The imaginary part of (7) together with (8) gives

$$\frac{\omega - \omega_0}{\omega} = -\omega \left(\delta \left\{ \frac{1}{2} \int_{\tau V} \epsilon |E_0|^2 \, d\tau_V \right\} \right) \Big/ (P_0 + P_V) Q_L. \qquad (9)$$

The varactor Q can be defined as

$$Q_V = \omega \left(\frac{1}{2} \int_{\tau V} \epsilon |E_0|^2 \, d\tau_V \right) \Big/ P_V$$

so that (9) can be written as

$$\frac{\delta \omega}{\omega} = \frac{\delta(P_V Q_V)}{(P_0 + P_V) Q_L} \left\{ 1 - \frac{P_V Q_V}{(P_0 + P_V) Q_L} \right\}^{-1}.$$

For typical experimental parameters

$$\frac{P_V Q_V}{(P_0 + P_V) Q_L} \ll 1$$

and the electronic tuning is calculated using (4).

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Computation of the Impedance of an Infinitely Long Helical Transmission Line by Numerical Methods

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***Abstract*—A numerical method is given for the determination of the impedance of an infinitely long thin-wire helix. The propagation constant of the current for zero tangential electric field is found and used in a variational expression for impedance. Asymptotic values of resistance versus pitch are compared with resistances of infinitely long straight-wire antennas.**

INTRODUCTION

Previous studies [2]–[7] of the propagation of waves on helices have been hindered by the complexity of the integrals occurring in expressions for electric-field intensity and input impedance. It has been necessary in the past to make many simplifying approximations from which it is possible to obtain much general information

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